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PROBABILITY DISTRIBUTIONS OF THE VELOCITY FLUCTUATIONS IN AXISYMMETRICAL TURBULENT WAKES

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Experimental data are reported on the one-dimensional probability distribution functions and up to the sixth statistical moments of the turbulent velocity fluctuations in hydrodynamic wakes of bluff and streamlined bodies. The data complement similar existing information for various turbulent flows: after a grid [1, 2]; in a two-dimensional wake [3]; in circular [4] and plane [5] jets; in a boundary layer [6]; in a circular pipe [7], etc. The problems of self-similarity of the investigated flow, the influence of the conditions of its evolution on the fluctuation characteristics in the self-similarity zone, and the role of intermittency at the wake boundary are discussed on the basis of the experimental data.

1. Experiments have been carried out in a low-turbulence wind tunnel with the application of a DISA Elektronik hot-wire anemometer system with a linearizer. Either a sphere of diameter $D = 1$ cm or a body of revolution (set up at zero angle of attack) with a midsection diameter $D = 1$ cm and an 8:1 elongation was suspended on wires of diameter 0.05 mm in the tunnel working section, which had a length of 4 m and a cross section of 40×40 cm and was fitted with triangular moldings in the corners to diminish secondary flows. In both cases the Reynolds number $\mathrm{Re} = U_\infty D/\nu = 10^4$ (where U_∞ is the freestream velocity and ν is the kinematic viscosity coefficient). Measurements have shown that this value of Re is large enough for the flow in the wake of the sphere to be self-similar with respect to the longitudinal coordinate and, hence, for similarity to hold with respect to the Reynolds number. To obtain similarity with respect to Re and self-similarity in the wake of the elongated body a turbulence generator in the form of a ring of diameter 8 mm and thickness 0.5 mm was set up in the bow region of the body. As a result, the drag forces F_x on the profiled body and the sphere did not differ appreciably, and so the drag coefficients c_x defined by the relation

$$F_x = c_x \rho S U_\infty^2/2, \quad S = \pi D^2/4,$$

were equal to 0.39 and 0.48 respectively. The small difference in the drag forces fit in quite well with one of the objectives of the experiments, which was to show that the characteristics of a wake in the self-similar region are not determined solely by the drag and freestream velocity, but depend strongly on the configuration of the body.

Below, we use a cylindrical coordinate system x, r, θ, which is attached to the body with its origin located at the trailing edge of the body and its x axis directed downstream. In addition to the constants U_∞ and D, we also use the following functions of x as typical scales of the velocity and length:

$$U_c(x) = U_\infty \left(\frac{x - x_0}{\sqrt{c_x S}} \right)^{-2/3}, \quad l_c(x) = \sqrt{c_x S} \left(\frac{x - x_0}{\sqrt{c_x S}} \right)^{1/3},$$

which are based on considerations of self-similarity of the flow. Here x_0 is the virtual origin of the wake and in the given experiments is close to zero for both bodies [8].

The probability density function p(e) of the stationary (in the statistical sense) hot-wire signal e(t) was estimated by means of an Intertechnique Histomat-S random-process analyzer. The signal e(t) was related to the longitudinal component of the velocity u(t) in the wake by the linear equation $e = a + ku$, where a and k are constants determined in static calibration of the hot-wire anemometer. The following statistical characteristics were determined in subsequent processing on a general-purpose computer: the probability density function of the velocity fluctuations

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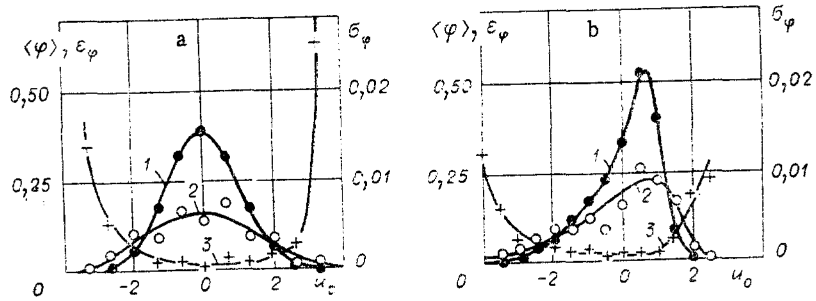

Fig. 1

$$f(u) = p(e)\left|\frac{de}{du}\right| = kp(a + ku);$$

the statistical moments

$$U = \int_{-\infty}^{\infty} uf(u)\,du, \quad \sigma^2 = \int_{-\infty}^{\infty} (u - U)^2 f(u)\,du,$$

$$\mu_n = \int_{-\infty}^{\infty} (u - U)^n f(u)\,du, \quad n = 3, 4, 5, 6;$$

the density function of the centered and normalized fluctuations

$$\varphi(u_0) = \sigma f(\sigma u_0), \; u_0 = (n - U)/\sigma$$

the characteristic function

$$\psi(is) = \int_{-\infty}^{\infty} \varphi(u_0)\, e^{isu_0} du_0, \quad i = \sqrt{-1}.$$

The average velocity and intensity of the fluctuations were also estimated independently by appropriate time averaging of realizations of u(t).

2. To investigate the plausibility of the conclusions drawn in the present study we have carried out a theoretical (by algorithms described, for example, in [9]) and experimental analysis of the measurement errors associated with the finite sizes of the statistical samples, time and level quantization of the signals, uncertainty in the placement of the hot-wire probe at a specified point of the flow (x, r), etc. The experimental procedure for estimating the errors entailed performing multiple repeated measurements under identical conditions at a series of characteristic points (x, r). The variance σ_Q^2 and coefficient of variation ε_Q of the error in a particular probability characteristic Q was calculated according to the formulas

$$\sigma_Q^2 = \frac{1}{N-1}\sum_{j=1}^{N} (Q_j - \langle Q\rangle)^2, \quad \varepsilon_Q = \frac{\sigma_Q}{\langle Q\rangle},$$

where Q_j is the measurement result in the j-th test; ⟨Q⟩ is the arithmetic mean of the results for N repeated measurements. As an illustration, Fig. 1 gives data on ⟨φ⟩, σ_φ, ε_φ obtained by this procedure for the probability density function Q = $\varphi(u_0)$ with N = 12 in the cross section x/D = 100 of the wake of the elongated body at the points r/l_c = 0 (Fig. 1a) and r/l_c = 0.4 (Fig. 1b): 1) ⟨φ⟩; 2) σ_φ; 3) ε_φ.

It must be noted that reasonably large samples were used in the given experiments, containing more than $2\cdot10^6$ discrete values of the signal e(t) recorded with a constant increment $\Delta t = 10^{-4}$ sec. The signal was level-quantized into 256 equal intervals. This ensured a small absolute error σ_φ. As for the relative error ε_φ, the signal e(t) always contains occasional large excursions, which have a very low probability and for which even such a large sample is inadequate. This fact is mirrored in the behavior of curves 3 in Fig. 1.

Additive noise is always present in the analyzed electrical signal e(t) as a result of electronic noise in the apparatus and ambient freestream turbulence in the wind tunnel. This was true of the present work, and special investigations showed that electronic noise accounted for the bulk of the signal noise (up to 70% in intensity). The role of the noise

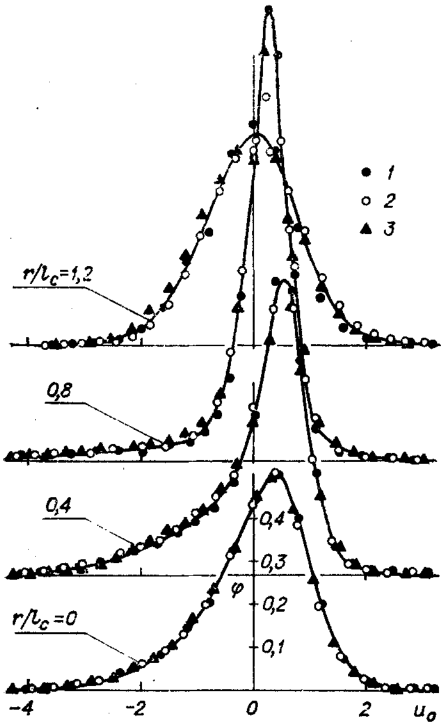

Fig. 2

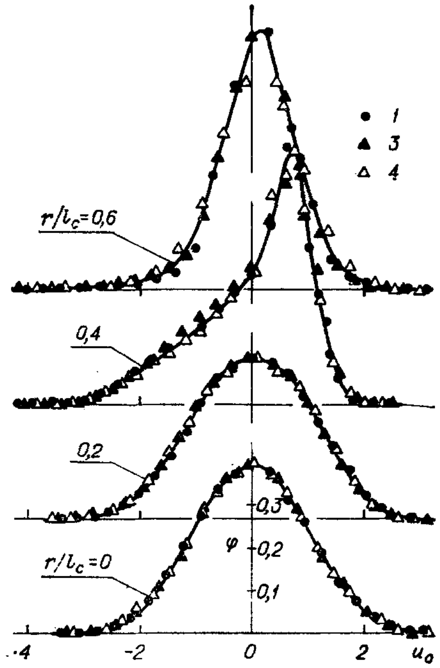

Fig. 3

was inconsequential in the vicinity of the wake axis, because the useful signal exceeded it tenfold in intensity. On approaching the boundary of the wake, however, the relative contribution of the noise increases, so that beyond the limits of the wake the hot-wire signal represents pure noise.

Let $e_0(t)$ be the useful signal of interest, and $e_*(t)$ the noise function. Assuming that the noise is additive and does not depend on the useful signal, as is certainly true if e_* is electronic noise and is acceptable as a first approximation if e_* represents ambient turbulence, we obtain the familiar relation for the density functions:

$$p(e) = \int_{-\infty}^{\infty} p_0(e_0)\, p_*(e_0 - e)\, de_0,$$

where p_0 and p_* are the characteristics of the useful signal and the noise. With the introduction of a correction the problem is to determine p_0 when p and p_* are known. Unfortunately, however, this mathematical problem is ill-posed in the sense that slight variations in the input data will produce large errors in the results of the calculations.

Attempts to solve this problem by regularization leads to the conclusion that it is better in tests of this nature to leave $p(e)$ uncorrected, rather than to distort the information by trying to solve an ill-posed problem. Results in which noise effects were found to be appreciable will be discussed below.

In regard to estimation of the statistical moments, the problem of correcting for noise effects is well-posed, and the experimental data used below are given with a correction according to the algorithms

$$U_0 = U - U_*,\ \sigma_0^2 = \sigma^2 - \sigma_*^2,\ \mu_3^0 = \mu_3 - \mu_3^*,$$
$$\mu_4^0 = \mu_4 - \mu_4^* - 6\sigma_0^2\sigma_*^2,\ \mu_5^0 = \mu_5 - \mu_5^* - 10\sigma_*^2\mu_3^0 - 10\sigma_0^2\mu_3^*,$$
$$\mu_6^0 = \mu_6 - \mu_6^* - 15\sigma_*^2\mu_4^0 - 20\mu_3^*\,\mu_3^0 - 15\sigma_0^2\mu_4^*,$$

in which the asterisk designates noise characteristics obtained by analyzing the signal from a probe removed from the wake. These algorithms are valid for independent additive noise.

3. The experimental data on the density functions are given in Fig. 2 (for the wake of the sphere) and Fig. 3 (for the wake of the elongated body). The points are numbered as follows: 1) x/D = 100; 2) 150; 3) 200; 4) 250. The data for different points with the same

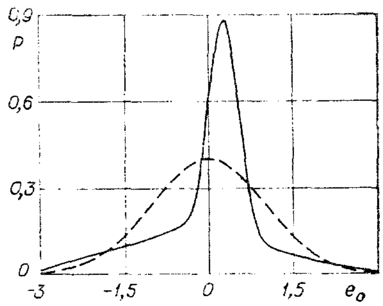

Fig. 4

value r/l_c =const are shifted vertically relative to one another by an arbitrary constant interval. For r/l_c =1.2 in the case of the sphere and r/l_c =0.6 in the case of the elongated body, the useful signal-to-noise intensity ratio is close to unity, and so the influence of noise is appreciable here. For smaller values of r/l_c indicated in the figures, the relative contribution of noise is negligible.

The data indicate that the functions $\varphi(u_0)$ are self-similar in the given interval of x/D, i.e., they depend only on r/l_c, but differently for each of the bodies. At like points of the wakes after the bodies with different configurations, the data differ considerably. This result is completely consistent with the analogous result obtained for other probability characteristics [8].

The density functions are not Gaussian at any point of the turbulent wake. For the sphere this fact is observed directly in Fig. 2, where the graphs of $\varphi(u_0)$ are skewed even on the wake axis. In the case of the elongated body, departures from a normal distribution near the wake axis appear only for the principal moments of even orders (see below). This conclusion is consistent with the result obtained for other turbulent flows.

The appreciable difference between the self-similar wakes of the sphere and the elongated body is attributable to the difference in the nature of the intermittence flow at the boundary of the wake [10], as exhibited by the fact that a probe located in the intermittency zone can be at times in a region of turbulent motion, at others in a region of nonturbulent motion, all in random fashion. As a result, the output signal is the logical sum of two random processes:

$$e(t) = e_1(t) \cup e_2(t),$$

where e_1 corresponds to fluctuations of turbulent origin, and e_2 to fluctuations of nonturbulent origin. Let us suppose that the probability of being in a turbulent region is γ (which by definition is the so-called intermittency factor) and that the probability of being in a nonturbulent region is $1 - \gamma$. These two events are incompatible and form a complete group. Then

$$p(e) = \gamma p_1(e) + (1 - \gamma) p_2(e),$$

where p_1 and p_2 are the density functions of the signals e_1 and e_2.

Let each of the signals e_1 and e_2 have a normal distribution function:

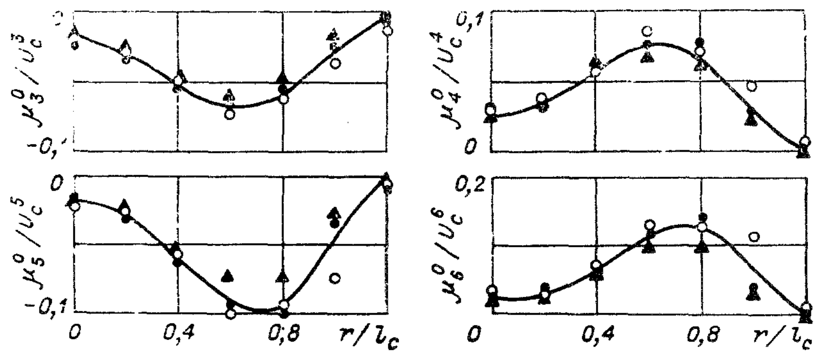

Fig. 5

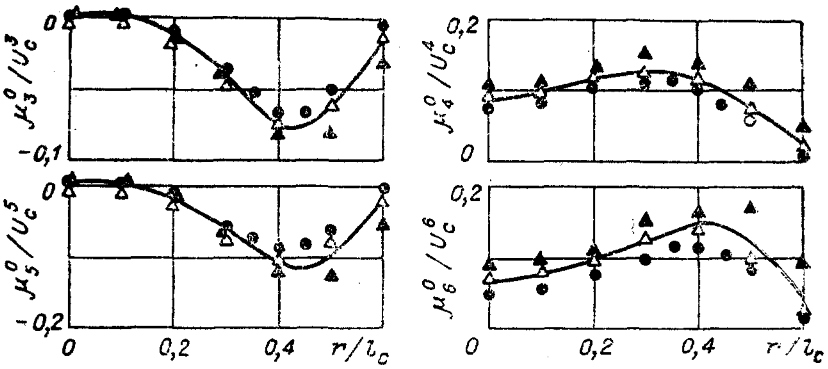

Fig. 6

$$p_i(e) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left\{-\frac{1}{2}\left(\frac{e-\langle e_i\rangle}{\sigma_i}\right)^2\right\}, \quad i=1, 2,$$

but let there be a difference between them in at least one of the parameters $\langle e_i\rangle$ or σ_i. Then p is no longer normal. As an illustration, Fig. 4 shows the result of a calculation for $\gamma = 0.5$, $\langle e_1\rangle = -2$, $\langle e_2\rangle = 0$, $\sigma_1 = 5$, $\sigma_2 = 1$, the solid curve representing the normalized [e_0 is related to e by the expression $e_0 = (e - \langle e\rangle)/\sigma)$] density function of the sum signal. A very pronounced difference is observed between this distribution function and the Gaussian function represented by the dashed curve with parameters $\langle e\rangle$ and σ calculated according to the relations

$$\langle e\rangle = \gamma\langle e_1\rangle + (1-\gamma)\langle e_2\rangle,$$
$$\sigma^2 = \gamma\sigma_1^2 + (1-\gamma)\sigma_2^2 + \gamma(1-\gamma)(\langle e_1\rangle - \langle e_2\rangle)^2.$$

This example, taken in conjunction with the experimental fact that the distributions of the fluctuations deviate from a Gaussian law even in turbulent flows lacking a clear-cut intermittency, suggests that not only simple summation, but also logical summation of turbulent eddy formations of different scales should be used in describing turbulence.

Experimental data on the statistical moments from the third through the sixth are shown in Fig. 5 for the wake of the sphere and in Fig. 6 for the wake of the elongated body. Fluctuations that occur infrequently but with large absolute values provide a significant contribution to these moments. We retain the same nomenclature as in Figs. 2 and 3 and use the scales U_c and l_c for normalization. In this normalization the experimental points for different values of x/D should lie on a single curve if the flow is self-similar also with respect to these probability characteristics. The self-similarity hypothesis is not refuted within the experimental error limits. Experimental data for the lower moments U and σ in the investigated flow are given in [8]. A test of the statistical hypothesis that the probability

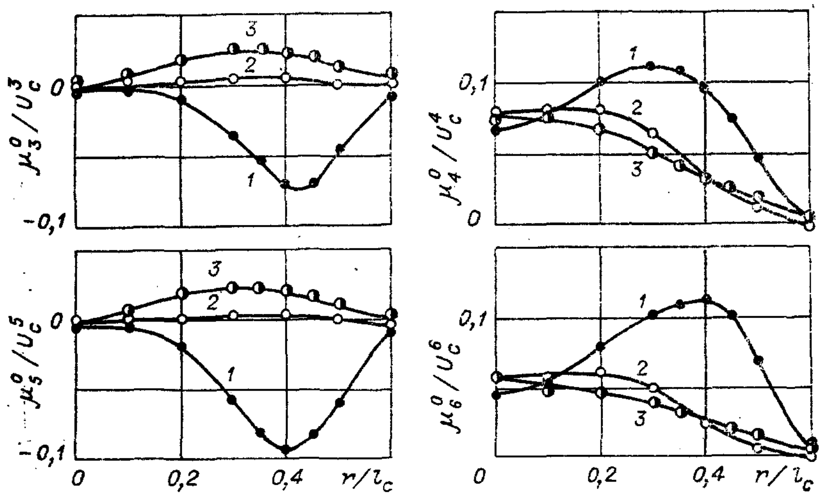

Fig. 7

distributions are Gaussian on the information obtained about the even-order moments show that it is inconsistent even for points on the wake axis after the elongated body, where the distributions themselves are symmetric.

Figure 7 shows data on the higher moments for all three components of the velocity vector in the wake of the elongated body for x/D = 100. The curves are numbered according to the components: 1) longitudinal; 2) radial; 3) tangential. On the wake axis the characteristics of the radial and tangential components should coincide, as was indeed confirmed in the experiments. These components are close to one another and do not deviate as much from a normal law as the longitudinal component or as at other points of the wake cross section.

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